

Problem 2.38

[Difficulty: 2]

2.38 The variation with temperature of the viscosity of air is correlated well by the empirical Sutherland equation

$$\mu = \frac{bT^{1/2}}{1 + S/T}$$

Best-fit values of b and S are given in Appendix A for use with SI units. Use these values to develop an equation for calculating air viscosity in British Gravitational units as a function of absolute temperature in degrees Rankine. Check your result using data from Appendix A.

Given: Sutherland equation with SI units

Find: Corresponding equation in BG units

Solution:

Governing equation:
$$\mu = \frac{b \cdot T^{\frac{1}{2}}}{1 + \frac{S}{T}} \quad \text{Sutherland equation}$$

Assumption: Sutherland equation is valid

The given data is $b = 1.458 \times 10^{-6} \cdot \frac{\text{kg}}{\text{m} \cdot \text{s} \cdot \text{K}^{\frac{1}{2}}}$ $S = 110.4 \cdot \text{K}$

Converting constants $b = 1.458 \times 10^{-6} \cdot \frac{\text{kg}}{\text{m} \cdot \text{s} \cdot \text{K}^{\frac{1}{2}}} \times \frac{\text{lbf}}{4.448 \cdot \text{kg}} \times \frac{\text{slug}}{32.2 \cdot \text{lbf}} \times \frac{0.3048 \cdot \text{m}}{\text{ft}} \times \left(\frac{5 \cdot \text{K}}{9 \cdot \text{R}} \right)^{\frac{1}{2}} b = 2.27 \times 10^{-8} \cdot \frac{\text{slug}}{\text{ft} \cdot \text{s} \cdot \text{R}^{\frac{1}{2}}}$

Alternatively $b = 2.27 \times 10^{-8} \cdot \frac{\text{slug}}{\text{ft} \cdot \text{s} \cdot \text{R}^{\frac{1}{2}}} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} b = 2.27 \times 10^{-8} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2 \cdot \text{R}^{\frac{1}{2}}}$

Also $S = 110.4 \cdot \text{K} \times \frac{9 \cdot \text{R}}{5 \cdot \text{K}} S = 198.7 \cdot \text{R}$

and
$$\mu = \frac{b \cdot T^{\frac{1}{2}}}{1 + \frac{S}{T}} \quad \text{with } T \text{ in Rankine, } \mu \text{ in } \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$$

Check with Appendix A, Table A.9. At T = 68 °F we find

$$T = 527.7 \cdot R$$

$$\mu = 3.79 \times 10^{-7} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$$

$$\mu = \frac{2.27 \times 10^{-8} \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2 \cdot R} \times (527.7 \cdot R)^{\frac{1}{2}}}{1 + \frac{198.7}{527.7}}$$

$$\mu = 3.79 \times 10^{-7} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$$

Check!

At T = 200 °F we find

$$T = 659.7 \cdot R$$

$$\mu = 4.48 \times 10^{-7} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$$

$$\mu = \frac{2.27 \times 10^{-8} \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2 \cdot R} \times (659.7 \cdot R)^{\frac{1}{2}}}{1 + \frac{198.7}{659.7}}$$

$$\mu = 4.48 \times 10^{-7} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$$

Check!